

Game Theory

What is Game Theory?

It is not (just) about...



What is Game Theory?

It is about:

- Making decisions
- Studying conflict and cooperation between intelligent decision makers

What does this have to do with games?!

- Situations are modelled as "games"
- Players are rational decision makers
- Each player has a set of actions they can do
- Everyone wants to maximize their personal satisfaction

That sounds boring...

But it isn't! Game Theory is used in:

- Economics - How much X should I produce

Game Theory and Oligopoly Behavior

Starbucks vs. San Francisco Coffee

The "players" are the firms: Two coffee shops, Starbucks and San Francisco Coffee.

The "moves" are the actions the firms can take: The coffee shops can either advertise around town or not advertise.

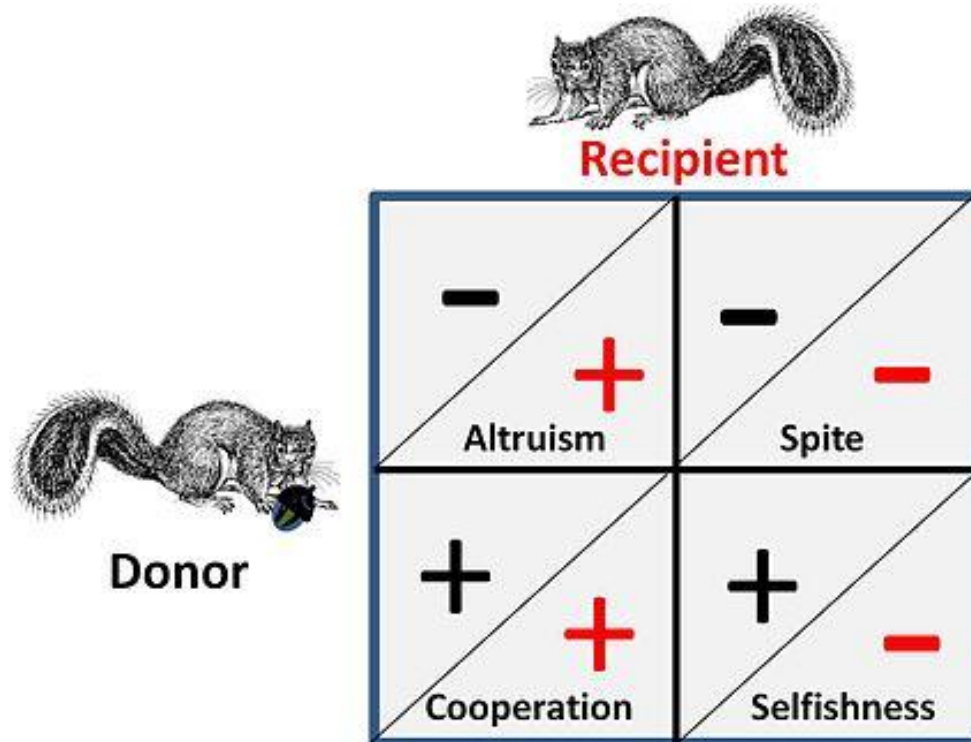
The "payoffs" are the profits the firms will earn: Advertising increases firms' costs, but can also increase revenues.

		Starbucks	
		don't advertise	advertise
SF Coffee	don't advertise	\$15 / \$15	\$20 / \$10
	advertise	\$20 / \$10	\$12 / \$12

That sounds boring..

But it isn't! Game Theory is used in:

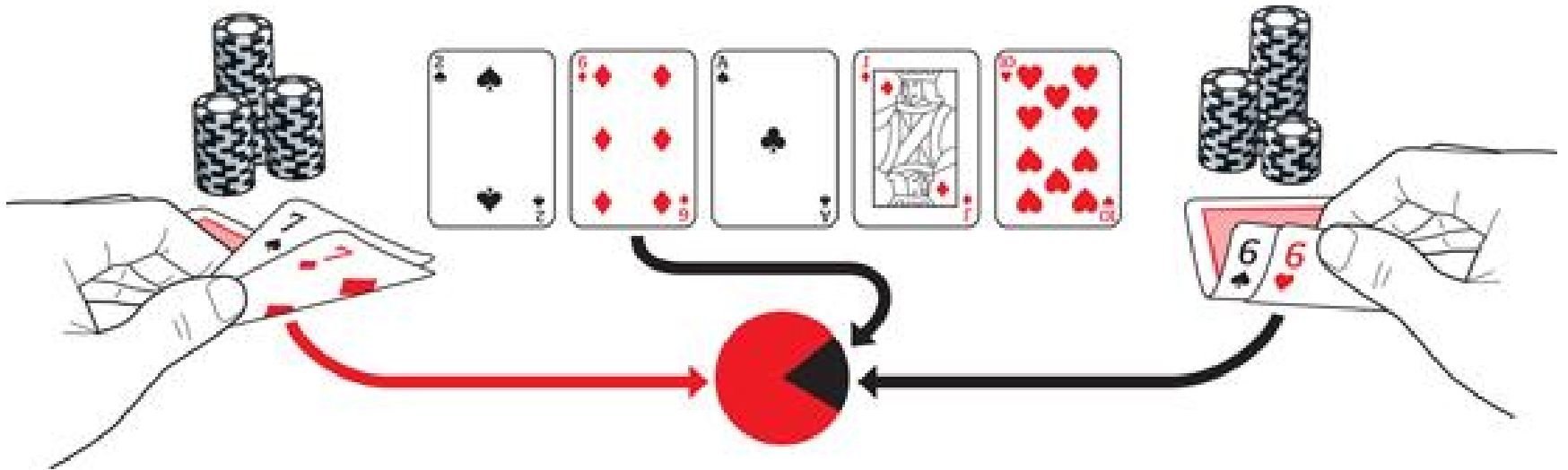
- Biology - Cooperative behaviour in nature!



That sounds boring..

But it isn't! Game Theory is used in:

- Other cool stuff



That sounds boring...

But it isn't! Game Theory is used in:

- Computer Science - We'll see soon

That sounds boring..

But it isn't! Game Theory is used in:

- Computer Science - We'll see soon
- Your love life?!



Game Theory + Your Love Life

Let's model Tinder as a "strategic form game"!

- 2 player game
- Each player can "Swipe Left" / "Swipe Right"
- If both players SR, everyone's happy!
- If you SR, but the other person SL, you just wasted a right swipe :(
- If both SL, we get a neutral outcome.

Game Theory + Your Love Life

Other person

	-	Swipe Right	Swipe Left
<i>You</i>	Swipe Right	5 , 5	-1 , 0
	Swipe Left	0 , -1	0 , 0





In this lecture...

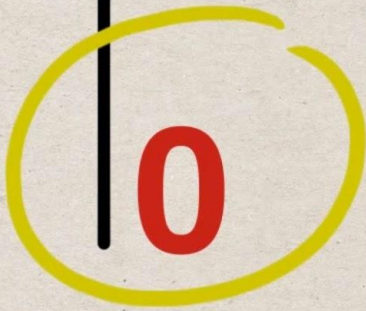
- Finding which decisions players will choose
- Analyzing strategies
- Networks
- Auctions (and how Google sells ads)
- Some other cool stuff we'd really like to cover in detail but we'll just talk about briefly

Prisoner's Dilemma







1.00

	 cooperate	 defect
 cooperate	1	0
 defect	0	2

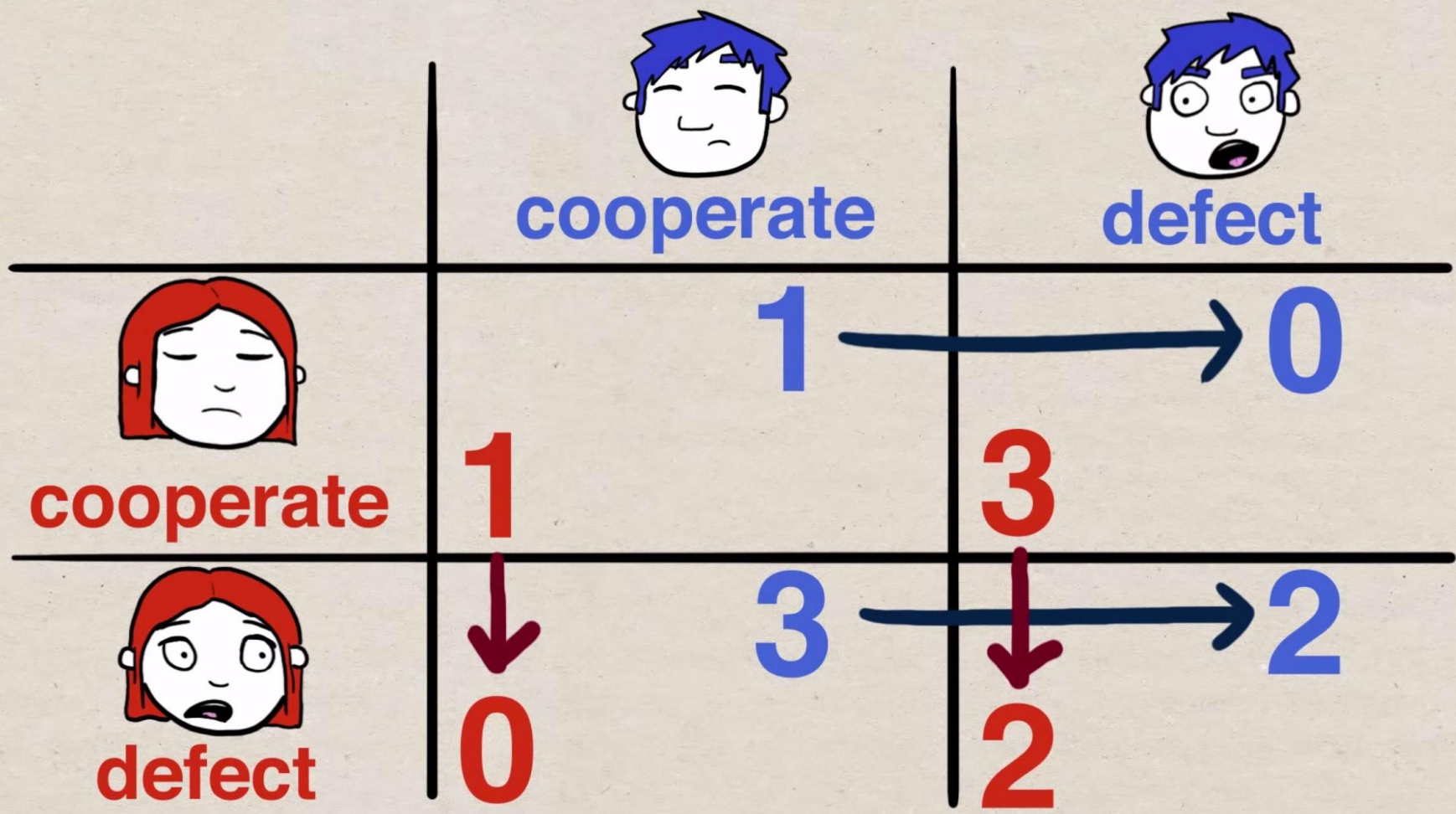


1.00

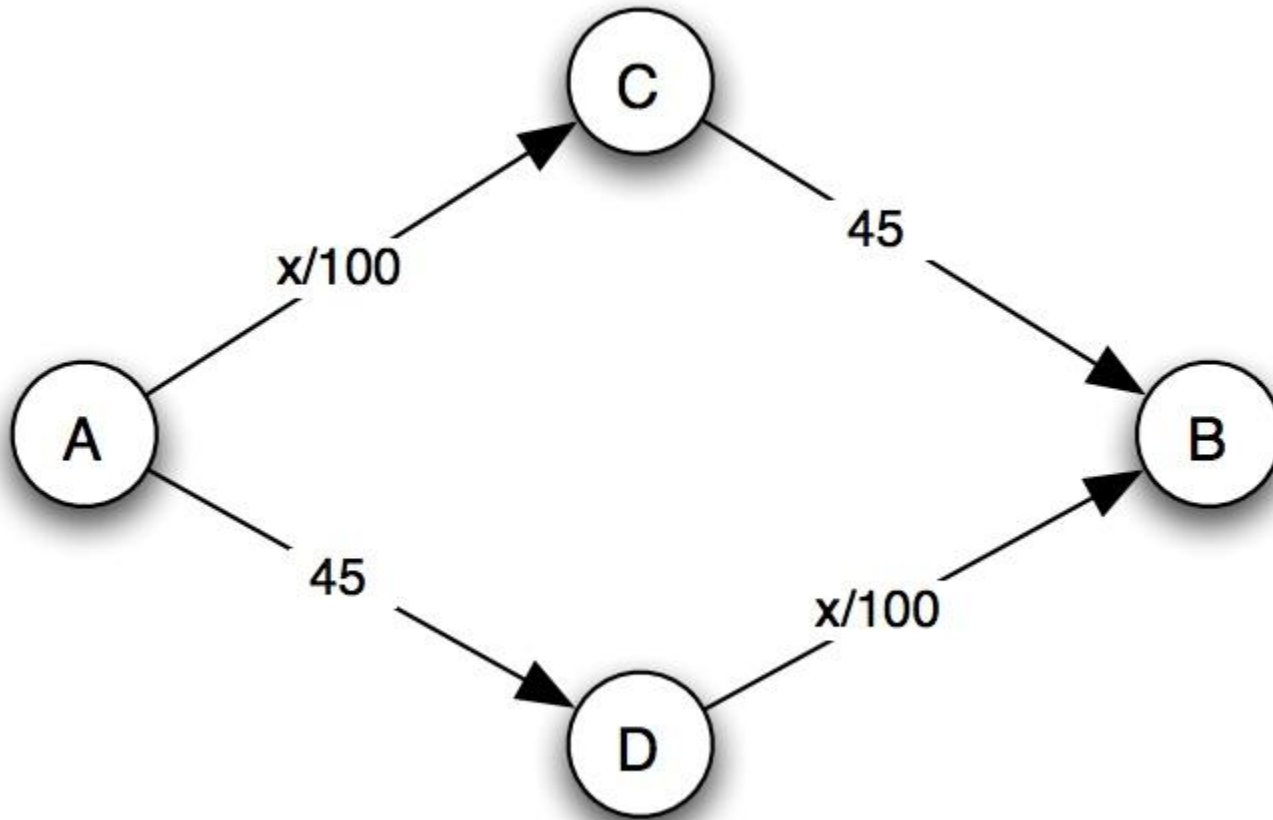
	 cooperate	 defect
 cooperate	1 1	0 3
 defect	3 0	2 2



1.00



Network Flow

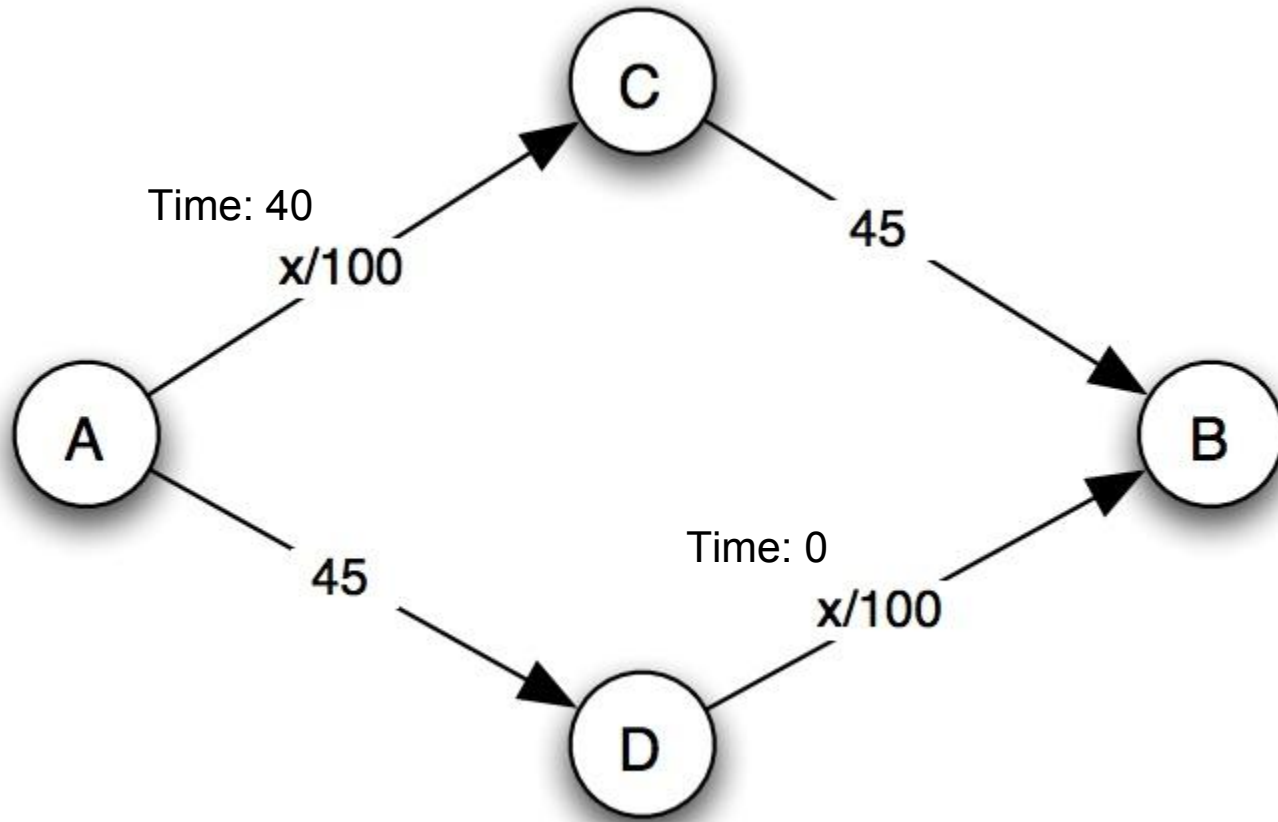


Nodes: Cities

Vertices: Highways

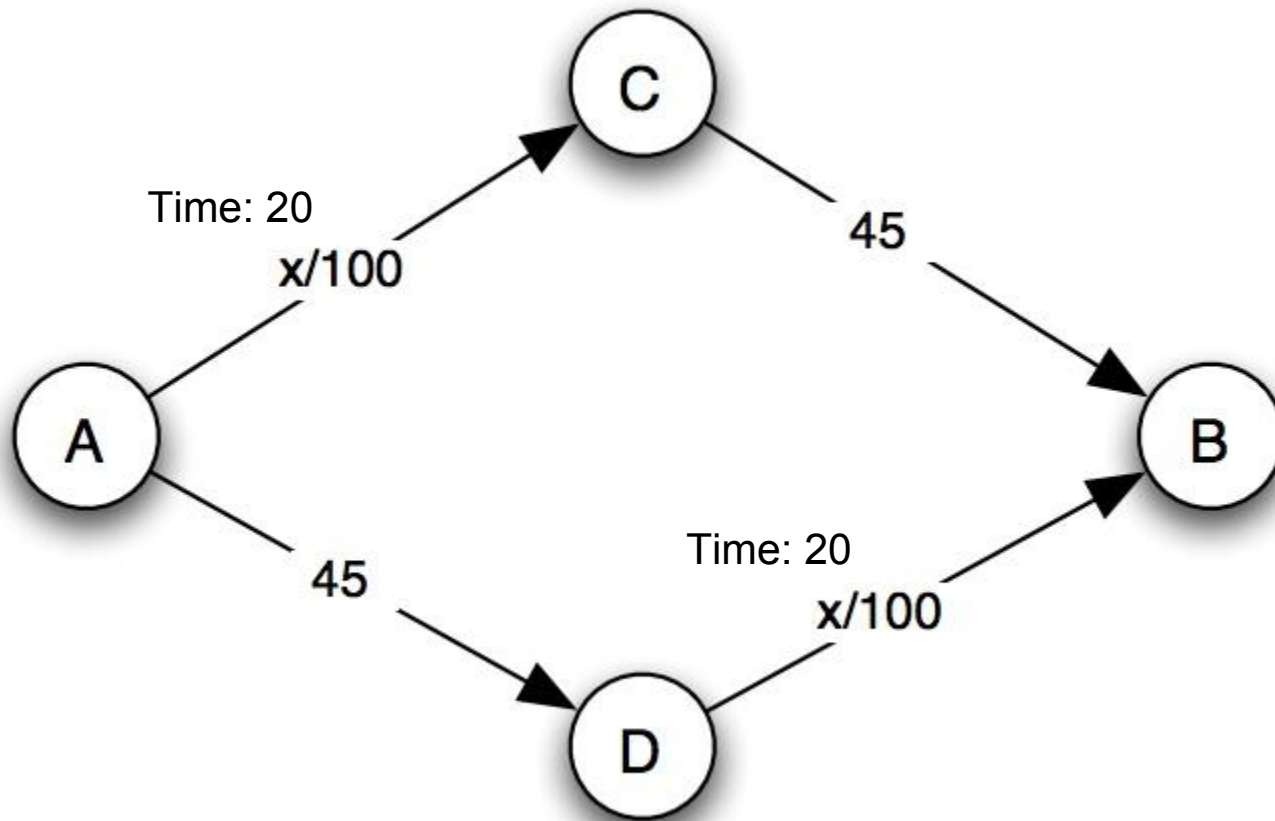
Situation: 4000 cars (players) choose a route (strategy) from A to B

Dominant strategy? Nash Equilibria?



Travelling Time: 85 through C, 45 through D

Expected Outcome: Drivers will start taking the route through D

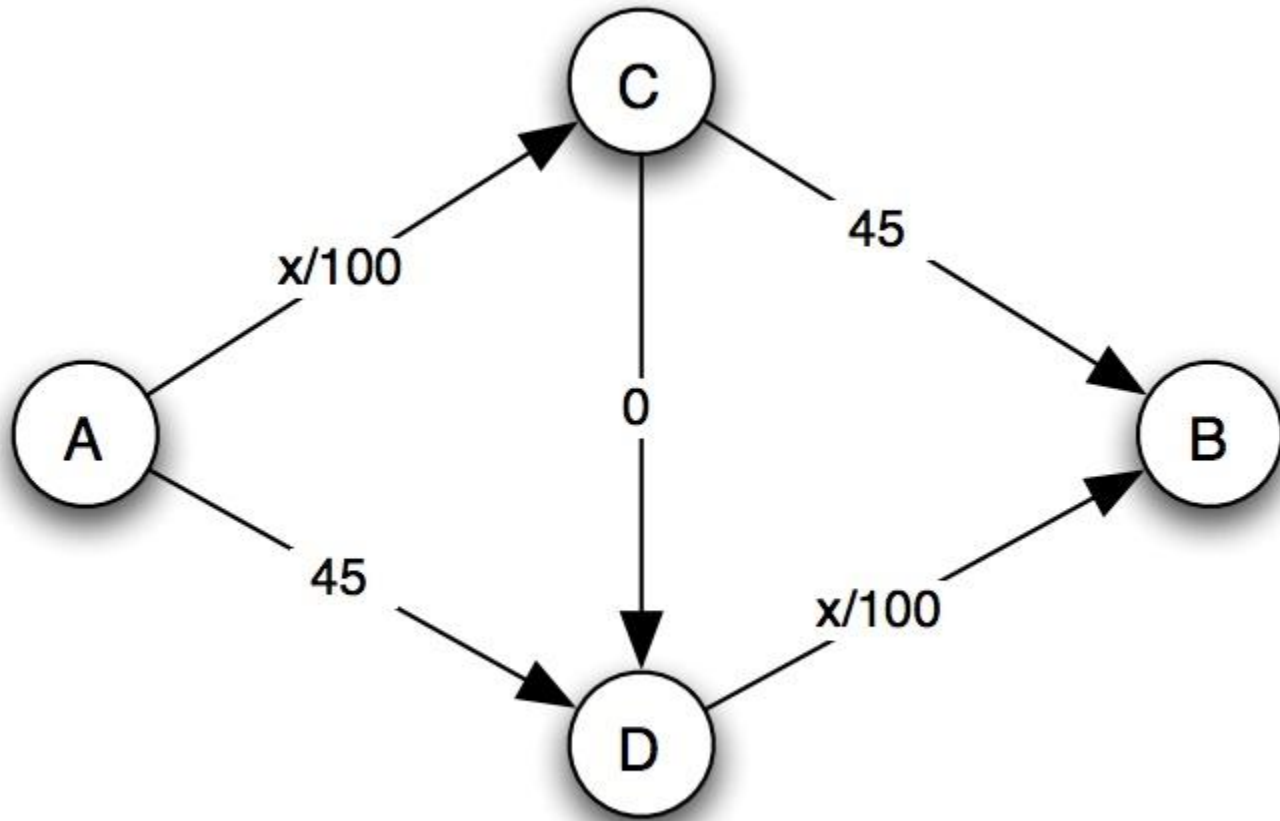


Expected Traffic Flow:

2000 drivers each side

Travelling time:

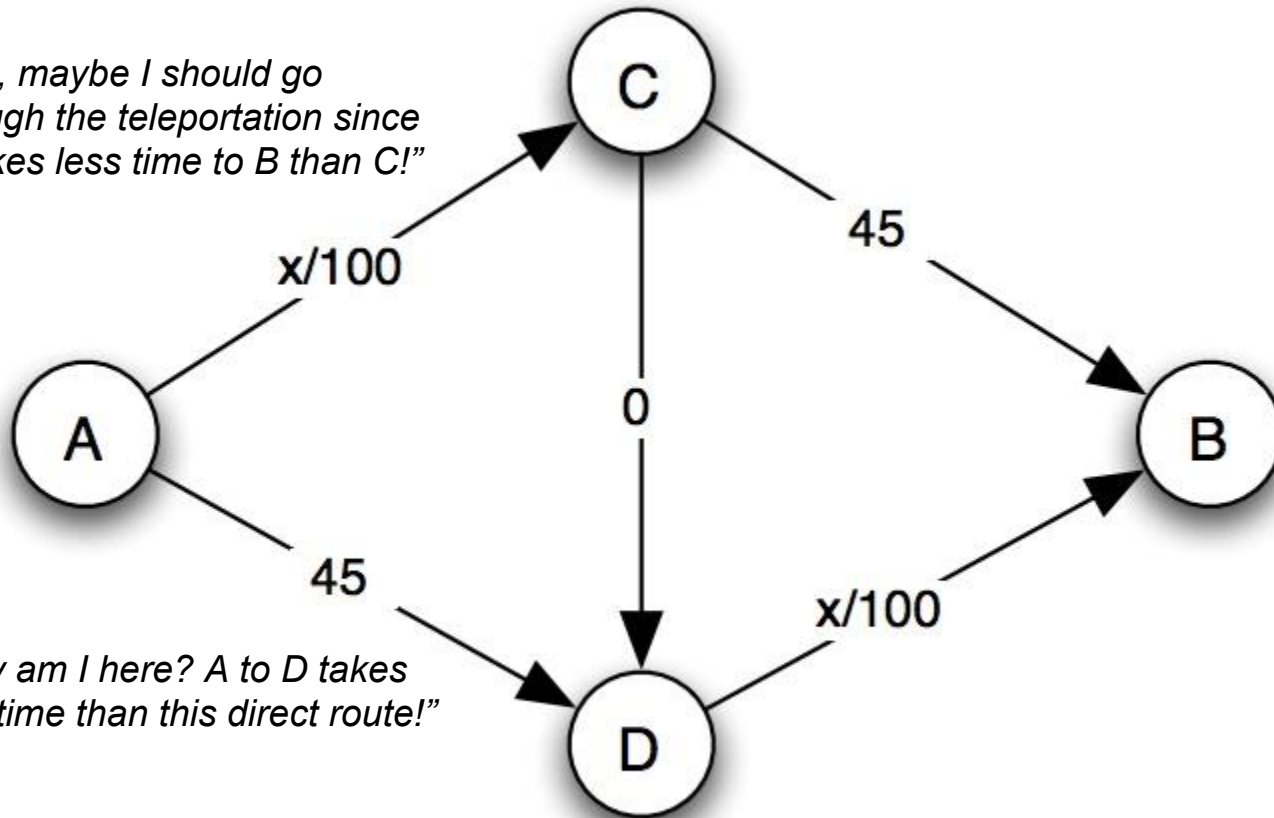
65 each direction



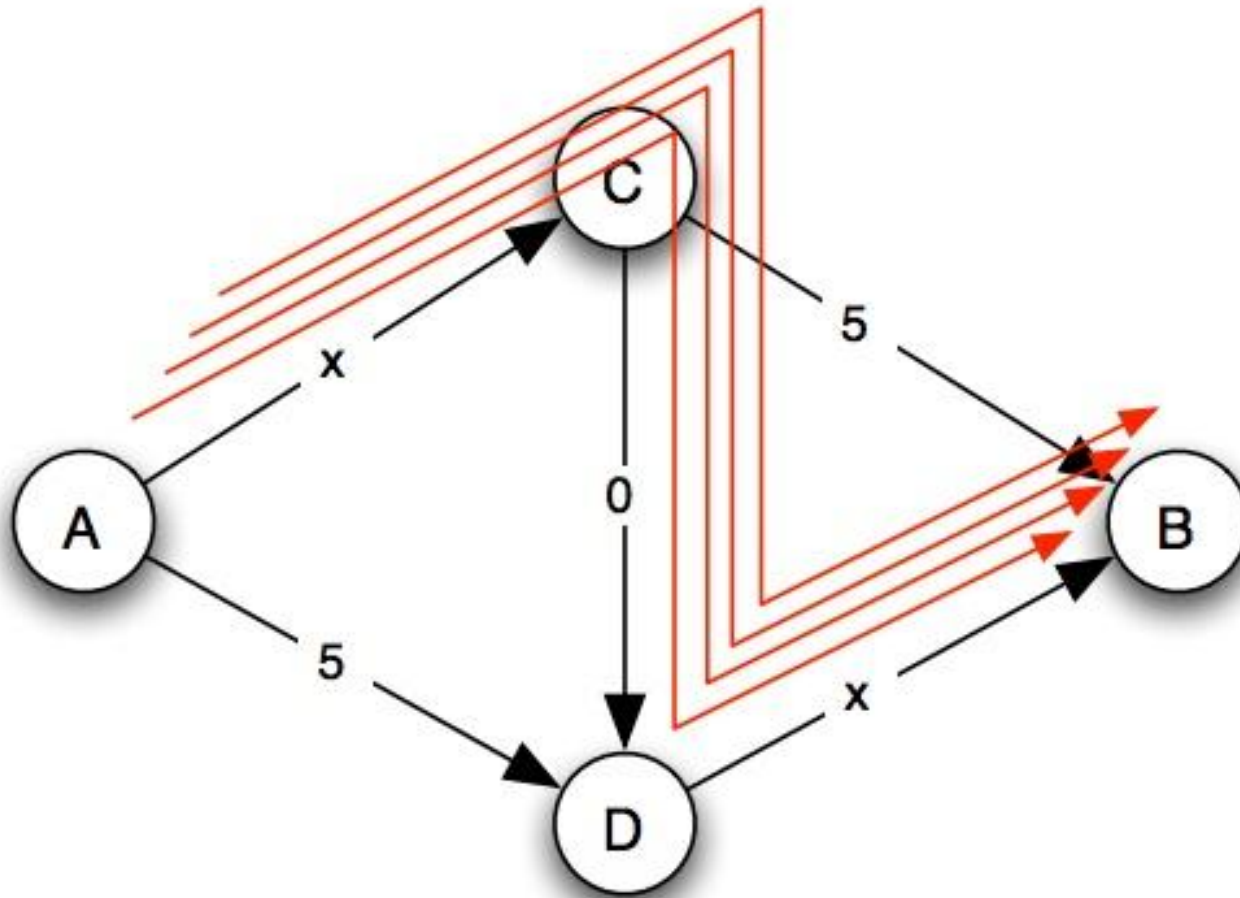
Change: Super fast highway/teleportation device from C to D

What is the impact? What is the new Nash equilibrium?

“Hey, maybe I should go through the teleportation since D takes less time to B than C!”

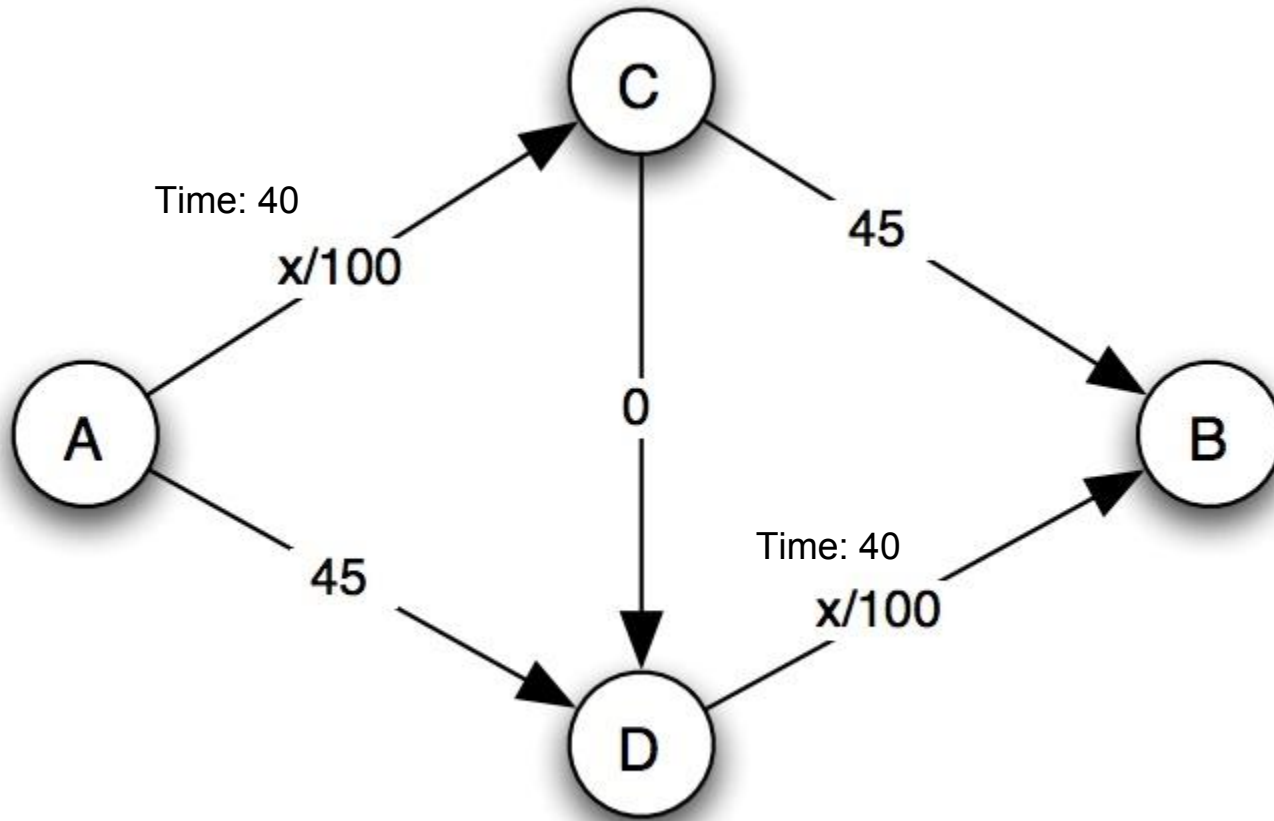


“Why am I here? A to D takes less time than this direct route!”



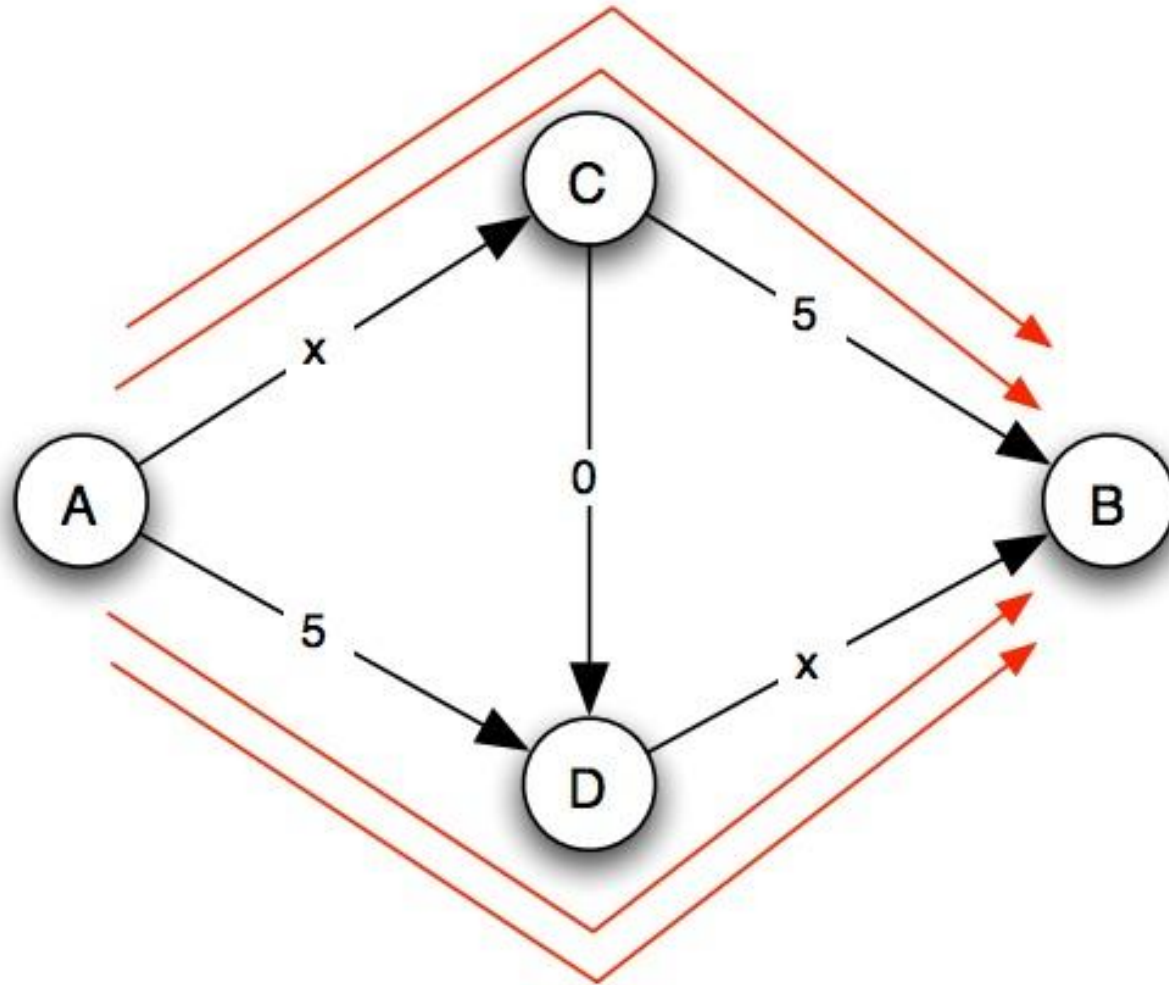
New traffic flow: From A through C, D to B

This is now the **dominant strategy**



Expected Travelling time: 80
Change: -15

Despite the improvement in transportation, we are worse off!



Socially optimum output != Nash equilibrium

Braess' Paradox

Adding resources to a transportation network can sometimes **hurt** performance at equilibrium levels

Network Latency



Auctions

Types of Auctions

- Ascending Bid (English Auctions)
- Descending Bid (Dutch Auctions)
- First + Second Price Sealed Bid Auctions

Auctions

- One seller
- Many buyers
 - Each buyer places a value on the item
- Seller does not know buyers' valuations
- Buyers do not know other buyers' valuations

Objectives

- Buyers
 - Maximize (PRICE - VALUE)
- Seller
 - Maximize PRICE
 - ???

Dutch Auction



Auction!

Price	Descending Bid	First Price Sealed Bid
100!		
90?		
80??		
70..		
60?! A: "I'll take it!"		A
50		
40	<i>B: Come on, just keep dropping</i>	B, D
30	<i>C: No way that thing is worth is 60. What value does that guy see in it?</i>	C
20		
10		

English Auction

Auctioneer

A (60)

B (40)

C (30)

“Let’s start the day
with 10, do I hear a
10?”

“10!”

“20”

“30”

“40”

“40 for this fine
specimen, just 4?
Do I hear a 50?”

*Come on, no one
else cares enough*

*I’m indifferent, with
or without it*

*I really wanted it, but
not for 40 :(*

“Sold!”

(Surplus +20)

(No benefit)

(No benefit)

Second Price Sealed Bid

A: 60

B: 40

C: 30

A wins, pays B's bid of 40

$$\text{Payoff for bidder } \begin{cases} v_i - \max_{j \neq i} b_j & \text{if } b_i > \max_{j \neq i} b_j \\ 0 & \text{otherwise} \end{cases}$$

Alternate bid b_i'

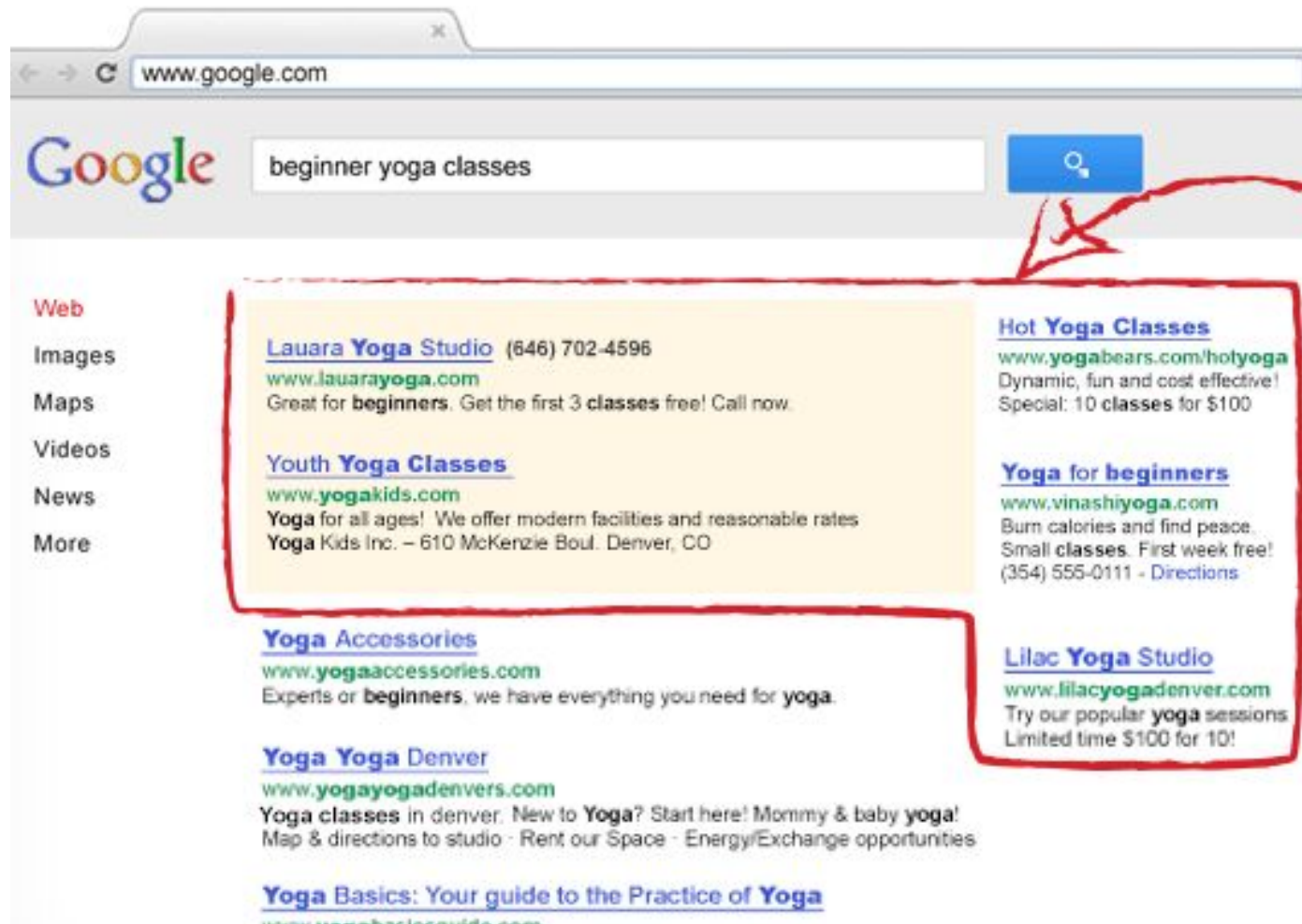
Raised bid affects outcome only if highest other bid b_j is in between. If so, i wins but pays more than value.

Truthful bid $b_i = v_i$

Lowered bid affects outcome only if highest other bid b_k is in between. If so, i loses when it was possible to win with non-negative payoff

Alternate bid b_i''

Sponsored Search (i.e. Ads)



A screenshot of a Google search results page for the query "beginner yoga classes". The browser's address bar shows "www.google.com". The search bar contains the text "beginner yoga classes" and a blue search button with a magnifying glass icon. On the left side, there is a vertical navigation menu with links for "Web", "Images", "Maps", "Videos", "News", and "More". The main content area displays several search results, with a red hand-drawn box highlighting a group of five sponsored ads. The ads are:

- Laura Yoga Studio** (646) 702-4596
www.lauarayoga.com
Great for **beginners**. Get the first 3 classes free! Call now.
- Youth Yoga Classes**
www.yogakids.com
Yoga for all ages! We offer modern facilities and reasonable rates
Yoga Kids Inc. – 610 McKenzie Boul. Denver, CO
- Hot Yoga Classes**
www.yogabears.com/hotyoga
Dynamic, fun and cost effective!
Special: 10 classes for \$100
- Yoga for beginners**
www.vinashiyoga.com
Burn calories and find peace.
Small classes. First week free!
(354) 555-0111 - [Directions](#)
- Lilac Yoga Studio**
www.lilacyogadenver.com
Try our popular yoga sessions
Limited time \$100 for 10!

Below the highlighted ads, there are three more search results:

- Yoga Accessories**
www.yogaaccessories.com
Experts or **beginners**, we have everything you need for yoga.
- Yoga Yoga Denver**
www.yogayogadenvers.com
Yoga classes in denver. New to **Yoga**? Start here! Mommy & baby yoga!
Map & directions to studio · Rent our Space · Energy/Exchange opportunities
- Yoga Basics: Your guide to the Practice of Yoga**
www.yogabasicsguide.com

Web 1.0 Advertising

- Advertisers paid per “impression”
- Non-targeted ads

Vox is brought to you by our sponsors

Find Your Graduating Class



I graduated in:

1997	▲
1987	▬
1977	▼

FIND

 classmates.com

The image shows a screenshot of a web advertisement for classmates.com. At the top, it says "Vox is brought to you by our sponsors". Below that is a large heading "Find Your Graduating Class" with a starburst graphic. Underneath the heading are three black and white portraits of young women. Below the portraits is a form with the text "I graduated in:" followed by a dropdown menu showing the years 1997, 1987, and 1977. To the right of the dropdown are up and down arrow buttons. Below the dropdown is a "FIND" button. At the bottom left is the classmates.com logo.

Targeted/Personalized Ads

The screenshot shows a Gmail interface in a Firefox browser window. The address bar displays the URL `https://mail.google.com/mail/?shva=1#inbox/135487d220d93466`. The Gmail header includes the search bar and navigation icons. The main content area shows an email from TripAdvisor with the subject "Your latest review has been published". The email body features a TripAdvisor advertisement with the headline "Come read your review on TripAdvisor" and a call to action: "Write just 1 more review and you'll earn your next badge. What else did you do in Bucharest, Romania? [Tell travelers about it.](#)". The advertisement also includes a rating of 4.0 stars and a link to "Tell travelers about it". On the right side of the email, there are additional targeted advertisements for "Rooms from EUR 27,50 p.P.", "Hotel Piazza di Spagna", and "Design Hotel in Liepaja".

Firefox

Gmail - Your latest review has been publ... +

https://mail.google.com/mail/?shva=1#inbox/135487d220d93466

+Mugur Gmail Calendar Documents Photos Sites Search More - Mugur Ionescu Share... Settings

Gmail

Mail

17 of 2,944

COMPOSE

Your home in Bucharest - www.imoteca.ro - Apartments & Villas for rent in the best areas of the city [Why this ad?](#)

Inbox

Important

Sent Mail

Drafts

Spam

Circles

[imap]/Sent

[imap]/Skickat

[imap]/Trash

Call log

Deleted Items

SMS (310)

More

Chat

Search people...

Your latest review has been published

TripAdvisor members@e.tripadvisor.com to me Feb 4 (5 days ago)

TripAdvisor

Add to circles

Show details

Ads - Why these ads?

Rooms from EUR 27,50 p.P.
Near Venice - Direct Bus
Albergo Campana, Dolo
www.albergoallacampagna.com

Hotel Piazza di Spagna
Boutique hotel in the heart of Rom
Book on-line now
www.hotelpiazzadispanna.it

Design Hotel in Liepaja
Centre of Liepaja - best rates.
Check our website for great deals.
www.promenadehotel.lv

tripadvisor

Hotels | Flights | Restaurants | Vacation Rentals | Trip Ideas

Come read your review on TripAdvisor

We've just published your review of Caru' cu Bere. Have a look.

Caru' cu Bere February 3, 2012

"Good tourist trap"

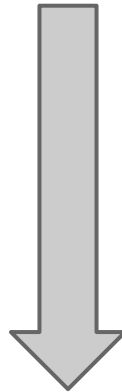
Write just 1 more review and you'll earn your next badge. What else did you do in Bucharest, Romania? [Tell travelers about it.](#)

Thanks,
The TripAdvisor Team

Charging Model Changed

Past:

Cost per impression (very low)



Present:

Cost per click/action (much higher)

Sponsored Search (i.e. Ads)

- Google auctions off the ad locations
- Vickrey-Clarck-Groves mechanism
- i.e. second price sealed bid
- Bidding your valuation is a dominant strategy

Sponsored Search (i.e. Ads)

- Bidders bid their real valuation
- Allows Google to know what advertisers' real valuations are

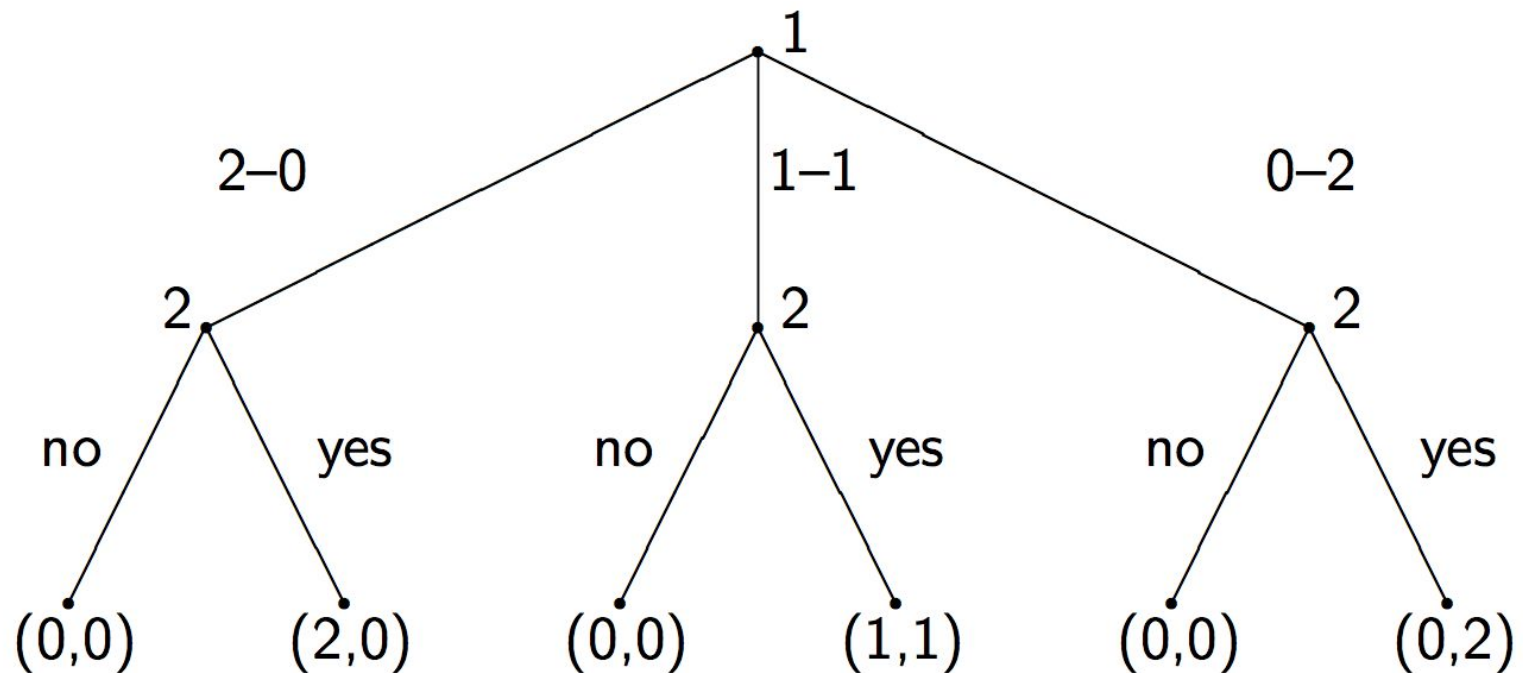
Other Cool Stuff

Isn't our definition of a game rather restrictive?

- Yes it is!
- Many other types of games exist
- Extended form games
- Repeated games

Extensive Form Games

- Players don't make decisions simultaneously



Extensive Form Games

Examples:

- Negotiations
- Chess (vs Rock-Paper-Scissors)

Repeated Games

- Repeating a strategic form game infinitely
- How are payoffs defined?
 - Average?
 - Limits?
 - Diminishing payoffs?

Mixed Nash Equilibria

- Strategies don't have to be just doing one action!
- We can randomize over multiple actions

Mixed Nash Equilibria

- No "pure" strategy can be at equilibrium
- 50% Heads, 50% Tails is an equilibrium

		Player B	
		Heads	Tails
Player A	Heads	1	-1
	Tails	-1	1

Cake cutting!

- Imagine there is a cake
- You have to share the cake with someone else
- How can we devise a way to split the cake between us?

Closing Remarks

- We've barely scratched the surface of game theory
- Game theory is very multi-disciplinary
 - Think of how it applies to your home disciplines

That's all folks!

:)